Ideas in Mathematics:
Angle Measurement

Supporting Teaching of Algebra: Individual Readiness

Key Concepts of Measuring Angles

- Angles are composed of two rays with a common vertex.
- Angles are measured by how widely or narrowly the two rays are spread apart or rotated around the vertex.
- Many students may struggle with measuring angles because one degree on a protractor is very small and hard for students to visualize.

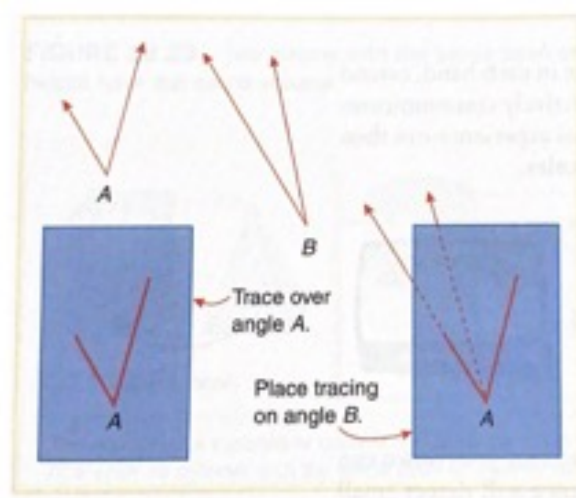
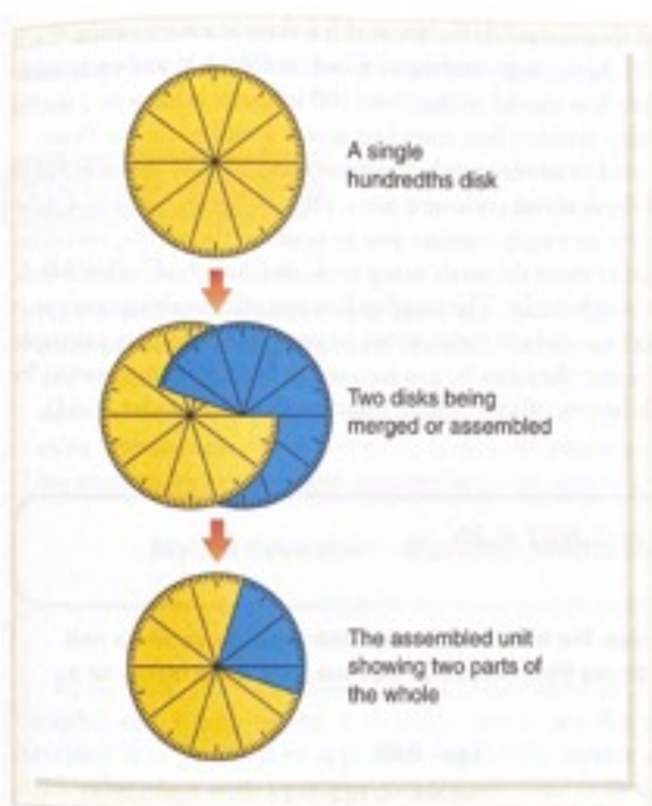


FIGURE 18.25 Which angle is larger?

(Van de Walle et al., 2019)



Teaching Angle Measurements

- Have students trace once angle and place it over a second angle of different spread. The rays should be of various lengths to emphasize the length of the rays does not impact the measure of an angle (See Figure 18.25).
- Have students create Angle Makers in which two different colored plates are cut and merged. Have students rotate the plates to estimate benchmark angles include 30, 45, 60, 90, 135, 180, and 270 degrees and then match observed angles (See Figure 16.5)
- Explicitly teach students how to use protractors and measure various angles using the tool.

Ideas in Mathematics:
Area

Supporting Teaching of Algebra: Individual Readiness

Key Concepts of Area

- Before teaching students to measure area, it is helpful to provide students with the opportunity to conceptually understand area.
- Many students confuse area and perimeter. After students conceptually understand area, explicitly teach perimeter and provide activities in which students must differentiate between the two concepts.

Teaching Area

- Provide opportunities for students to compare shapes in which the area is rearranged or in which the shapes share one common dimension or property (e.g., two rectangles with the same width). This activity can be extended using tangrams.
- Provide students with physical models to cover the area of shapes. Objects can include tiles, sticky notes, shapes on geoboards, floor tiles on the ground,
- Practice activities that develop students' understanding of area, as seen in Activity 18.17.

Activity 18.17 CCSS-M: 3.MD.C.5; 3.MD.C.6; 4.MD.A.3

Cover and Compare

STUDENTS with SPECIAL NEEDS

Give students the Two Rectangles, a Parallelogram, a Trapezoid and a Blob Activity Page where the areas are not the same but with no area that is clearly largest or smallest. Ask students to predict which shape has the smallest and the largest area and then use a strategy to compare the areas. Brainstorm strategies and create a list of options to support students with special needs. Students may trace or glue the same two-dimensional unit on the shapes, place tiles on them, or cut the shapes out and place them on grid paper.

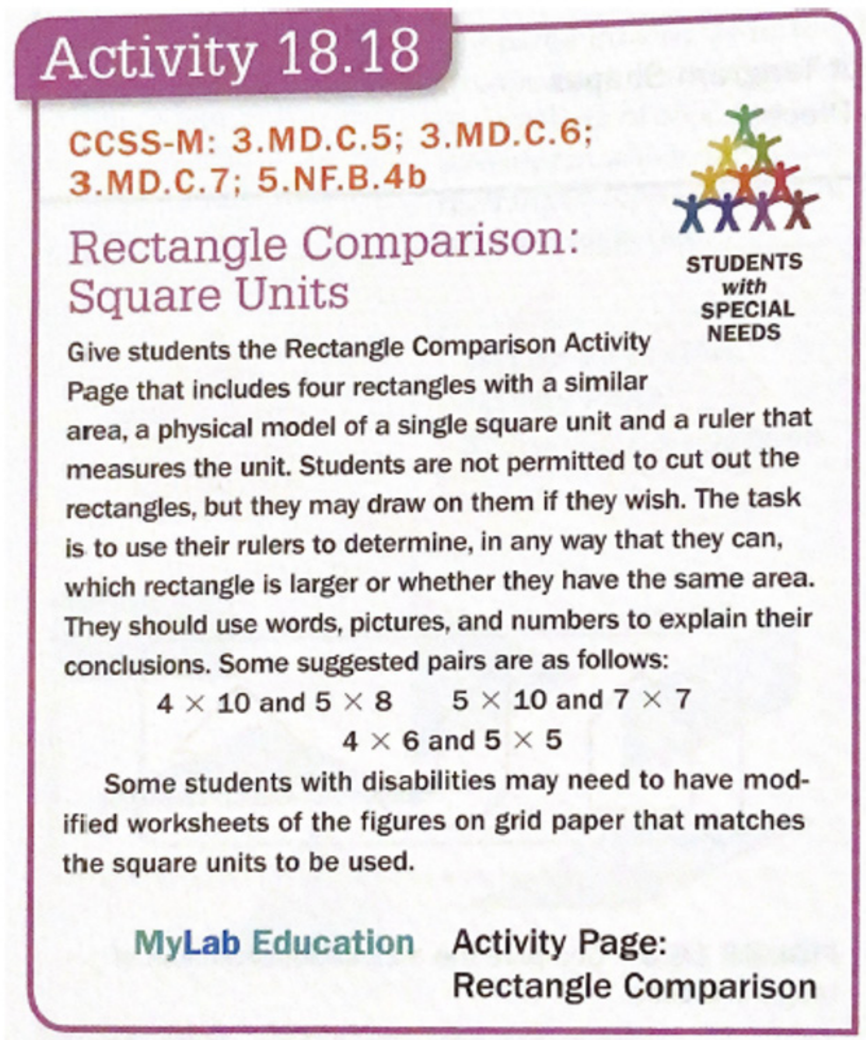
MyLab Education Activity Page: Two Rectangles, a Parallelogram, a Trapezoid, and a Blob

(Van de Walle et al., 2019)

Ideas in Mathematics:
Area

Readiness
Individual
Algebra:
Teaching of
Supporting

- Practice measuring area in square units, as seen in Activity 18.18.



Activity 18.18

CCSS-M: 3.MD.C.5; 3.MD.C.6; 3.MD.C.7; 5.NF.B.4b

Rectangle Comparison: Square Units

STUDENTS with SPECIAL NEEDS

Give students the Rectangle Comparison Activity Page that includes four rectangles with a similar area, a physical model of a single square unit and a ruler that measures the unit. Students are not permitted to cut out the rectangles, but they may draw on them if they wish. The task is to use their rulers to determine, in any way that they can, which rectangle is larger or whether they have the same area. They should use words, pictures, and numbers to explain their conclusions. Some suggested pairs are as follows:

4×10 and 5×8 $\quad$ 5×10 and 7×7

4×6 and 5×5

Some students with disabilities may need to have modified worksheets of the figures on grid paper that matches the square units to be used.

MyLab Education Activity Page: Rectangle Comparison

(Van de Walle et al., 2019)

- Introduce or practice the concept of multiplication as arrays to find the area of rectangles.

- Have students develop formulas for area, based on their conceptual understanding. If students struggle with this task, remind students of the steps they take to solve multiplication problems using an area without counting every square.

- Once students are comfortable with the area of rectangles, move towards solving the area of a parallelogram, similar to that shown in Activity 18.23.

Ideas in Mathematics:
Area

Supporting Teaching of Algebra: Individual Readiness

Activity 18.23 CCSS-M: 6.G.A.1

Area of a Parallelogram

Give students the Grid of Parallelograms Activity Page or, for a slightly harder challenge, drawn on plain paper with all dimensions—the lengths of all four sides and the height. Ask students to use what they have learned about the area of rectangles to determine the areas of these parallelograms. Students should find a method that will work for any parallelogram, even if not drawn on a grid.

MyLab Education Activity Page: Grid of Parallelograms

(Van de Walle et al., 2019)

• From parallelograms, students should be introduced to finding the area of triangles and trapezoids. Students can be supported by showing them ways that triangles and parallelograms form rectangles. In addition, trapezoids can be combined to make parallelograms, or trapezoids can be decomposed to rectangles and triangles. See Figure 18.14–18.16 for transforming shapes.

Ideas in Mathematics:

Area

Readiness
Individual
Algebra:
Teaching of
Supporting

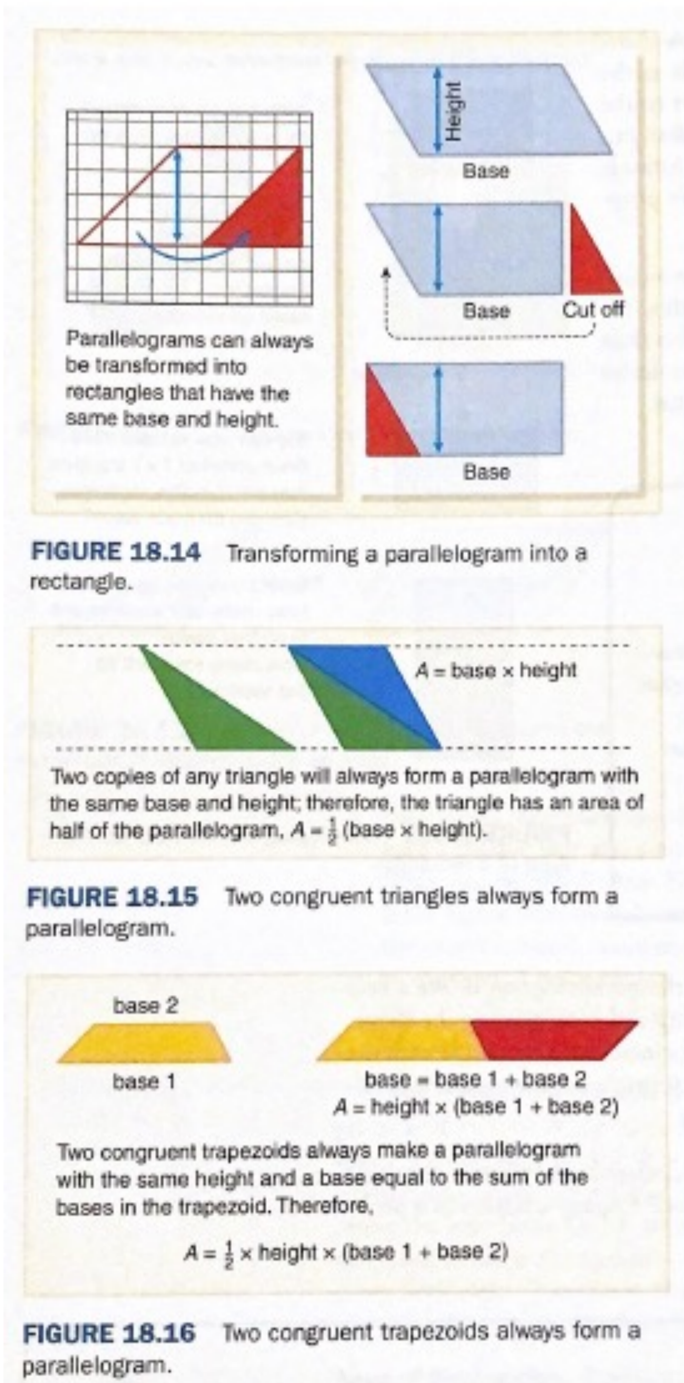


FIGURE 18.14 Transforming a parallelogram into a rectangle.

FIGURE 18.15 Two congruent triangles always form a parallelogram.

FIGURE 18.16 Two congruent trapezoids always form a parallelogram.

(Van de Walle et al., 2019)

To teach surface area, it is beneficial for students to use nets as visualization tools.

Ideas in Mathematics:
Volume

Supporting Teaching of Algebra: Individual Readiness

Key Concepts of Volume

- Liquid volume or capacity refers to the amount that a container will hold.
- Volume can refer to the capacity of an object as well as the amount of space occupied by the three-dimensional object.
- Surface area does not dictate volume, instead prisms and shapes with more cube-like dimensions have greater capacity than objects with the same surface area that are long and narrow.
- Students may struggle with identifying and using the height of an object to calculate the volume. Explicitly teach how to identify the height of an object.

Teaching Volume

- Younger students can sort objects to determine which objects "hold more" or have greater capacity. This can also be done by filling objects with beans, rice, water, or popcorn to determine the capacity of the objects.
- For older students, you can provide them with inch cubes or blocks and ask them to create prisms of various dimensions and ask them to calculate the surface area and volume for each prism.
- In addition, you can provide students with empty boxes and ask them to determine the volume, as demonstrated in Activity 18.29. It may be beneficial to have students note the number cubes that fit in the bottom of the box.

Activity 18.29 CCSS-M: 5.MD.C.3, 5.MD.C.4, 5.MD.C.5; 6.G.A.2; 7.G.B.6

Box Comparison: Cubic Units

Provide students with a pair of small boxes that you have made from card stock (see Figure 18.20). Use unit dimensions that match the cubes that you have for units. Students are given two boxes, one cube, and a corresponding ruler (If you use 2-centimeter cubes, make a ruler with 2-centimeter units). Ask students to decide which box has the greater volume or if they have the same volume.

Here are some suggested box dimensions ($L \times W \times H$):

$6 \times 3 \times 4$ $5 \times 4 \times 4$ $3 \times 9 \times 3$ $6 \times 6 \times 2$ $5 \times 5 \times 3$

Students should use words, drawings, and numbers to explain their conclusions. Repeat with boxes with fractional values. For example, ask students to estimate and then determine which of the following shipping boxes has the greatest and least volume:

$\frac{1}{4}$ ft $\times$ 3 ft $\times$ $2\frac{1}{2}$ ft $\quad$ 4 ft $\times$ $\frac{3}{4}$ ft $\times$ $\frac{1}{2}$ ft $\quad$ $\frac{5}{12}$ ft $\times$ 2 ft $\times$ $\frac{3}{4}$ ft $\quad$ 2 ft $\times$ $\frac{1}{4}$ ft $\times$ 3 ft

(Van de Walle et al., 2019)

Ideas in Mathematics:
Volume

Supporting Teaching of Algebra: Individual Readiness

- Once students are comfortable understanding the concept of volume, develop formulas for finding volume with students. Begin with regular prisms, and then move to cones, pyramids, and spheres. Utilize translucent plastic models and emphasize that a reoccurring theme in calculating volume is to find the base times the height.

Resources for Teaching Volume

- **Toy Theater Cube Activities**
  In this space, students can practice building digital three-dimension shapes with cubes.
- **NCTM Geometric Solids**
  This space allows students to explore digital three-dimensional objects.