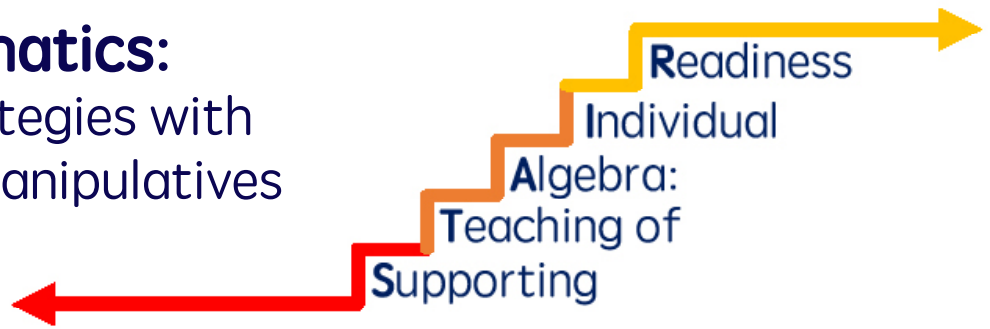


Ideas in Mathematics:

Paper & Pencil Strategies with Readily Available Manipulatives



Manipulatives are physical objects that can be used to explore mathematical concepts.

Fast Facts:

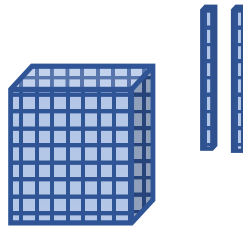
- Manipulatives are recommended for students of all ages (yes, even middle school!)
- Determined to be effective for students with and without disabilities
 - **Concrete manipulatives:** tangible objects used to build the foundational understanding
 - **Virtual manipulatives:** interactive, web-based representations of objects used to connect to the abstract

Tips for Use:

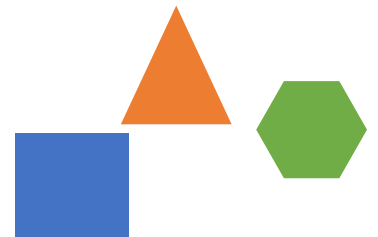
- Manipulatives should connect to the concept being taught and students' developmental level
- Instruction using manipulatives is an integral part of the concrete → semi-concrete → abstract sequence
- Manipulatives don't have to be expensive; many types of problems can be represented with manipulatives made from easily accessible materials!

Commonly Used Manipulatives:

Base Ten Blocks



Shape Tiles



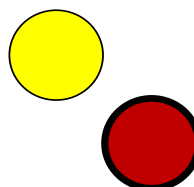
Beans & Cups



Algebra Tiles



Two-Colored Counters

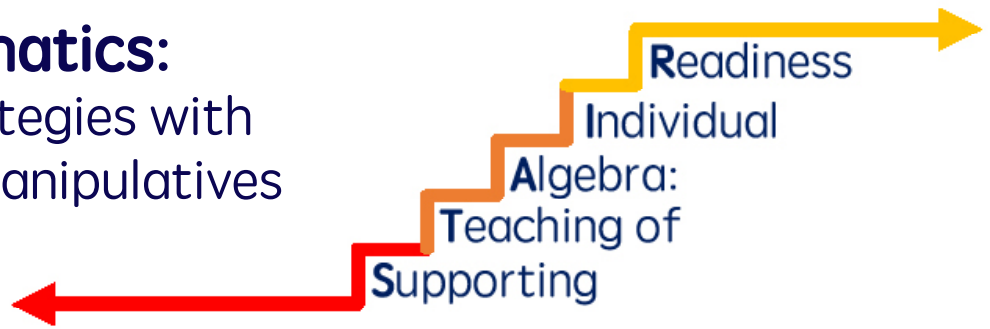


Unifix Cubes



Ideas in Mathematics:

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One way to utilize manipulatives is through
mathematical modeling!

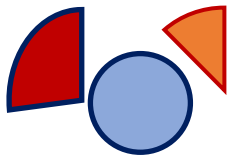
Three types of Models:

1. Area
2. Length of Measurement
3. Set

What types of manipulatives can be used with models?

Area Models

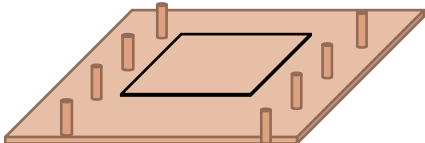
Circular pieces



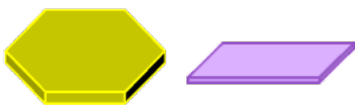
Rectangular regions



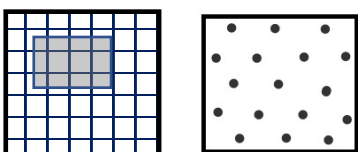
Geoboards



Pattern blocks

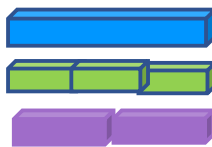


Grid or dot paper



Length Models

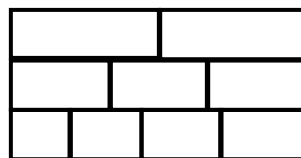
Fraction strips or
Cuisenaire Rods



Number lines

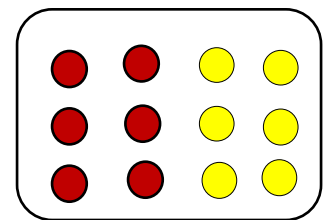


Folded paper strip
diagrams



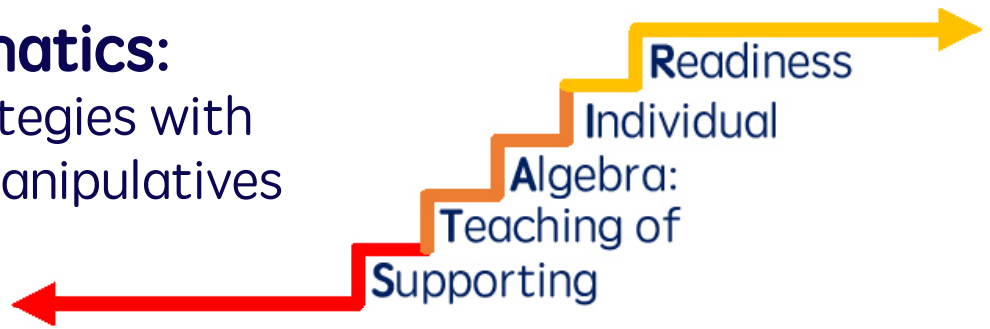
Set Models

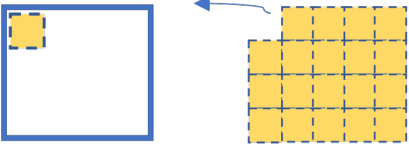
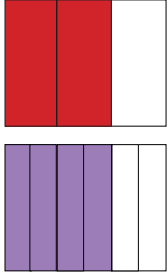
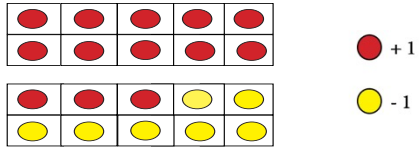
Two-colored
counters



Ideas in Mathematics:

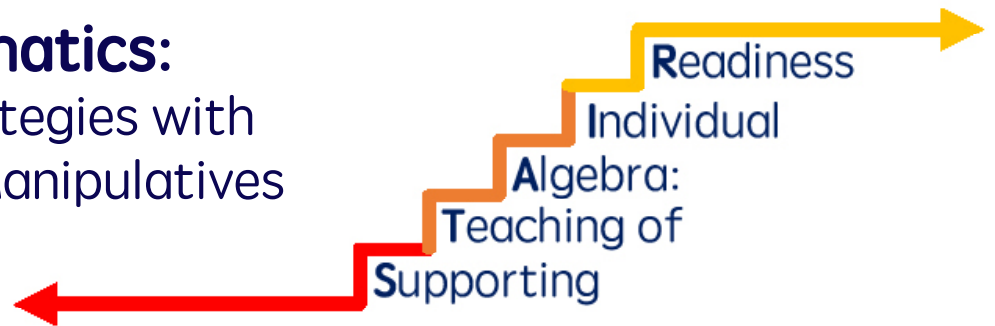
Paper & Pencil Strategies with Readily Available Manipulatives



Task	Manipulatives	Example Problem	How to Use
Measuring area and perimeter	<ul style="list-style-type: none"> - Card stock cut into squares of equal size - 1-inch square tiles 	Angie used tape to make a square. What is the area of the square?	<p>Use painter's tape to create a square on the floor or student desk.</p> <p>Have students fill the square with squares cut from cardstock.</p> <p>Using either the area formula or by adding up the squares inside the figure, students can solve.</p> <p><i>*Each card/tile represents 1 square unit.</i></p> 
Fraction equivalencies	<ul style="list-style-type: none"> - Fraction tiles or paper strips - Cuisenaire rods (if available) 	<p>Write an equivalent fraction to .</p> <p>Use your fraction strips to justify your answer.</p>	<p>Students can use the fraction strips (or Cuisenaire rods, if available) to represent the pair of equivalent fractions.</p> 
Adding or Subtracting integers	<ul style="list-style-type: none"> - Ten frames with two-colored counters 	<p>Represent the subtraction problem below and find the solution.</p> $13 - 8 =$ <hr style="width: 50px; margin-left: 0;"/>	<p>Point out that one red counter represents +1 and one yellow represents -1. Have students use the ten frames and counters to represent the subtraction problem.</p> 

Ideas in Mathematics:

Paper & Pencil Strategies with
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Implementation Resources:

IRIS Modules: <https://iris.peabody.vanderbilt.edu/module/math/cresource/q2/p05/>

National Library of Virtual Manipulatives: <http://nlvm.usu.edu/en/nav/vlibrary.html>